# Bearing Load Distribution Studies in a Multi Bearing Rotor System and a Remote Computing Method Based on the Internet 

Zhao Jian Yang*, Ze Jun Peng<br>College of Mechanical Engineering, Taiyuan University of Technology, Taiyuan, 030024, China<br>Seock Sam Kim<br>School of Mechanical Engineering, Kyungpook National University, Daegu, 702-701, Korea


#### Abstract

A model in the form of a Bearing Load Distribution (BLD) matrix in the Multi Bearing Rotor System (MBRS) is established by a transfer matrix equation with the consideration of a bearing load, elevation and uniform load distribution. The concept of Bearing Load Sensitivity (BLS) is proposed and matrices for load and elevation sensitivity are obtained. In order to share MBRS design resources on the Internet with remote customers, the basic principle of Remote Computing (RC) based on the Internet is introduced ; the RC of the BLD and BLS is achieved by Microsoft Active Server Pages (ASP) technology.


Key Words: Multi Bearing Rotor System, Bearing Load, Sensitivity, Internet, ASP, Remote Computing

## 1. Introduction

A bearing rotor structure is widely used in large machinery equipment. The MBRS is a typical form of large turbine-generator sets shown in Fig. 1. In the MBRS, several rotors are coupled into a series of shaft, which are fixed on different kinds of bearings. The rotor bearing system becomes redundant. The bearings support the static and dynamic load in the MBRS. The variations of BLD among the bearings are caused by journal fluctuation and variations of elevations on the bearings. These include installation elevation, base elevation as well as elevation increase caused by thermal expansion. In other words, any alteration of bearing elevation will change the BLD; hence, other properties of the bearing

[^0]load distribution and the dynamic characteristics of the rotor-bearing system. Unbalanced BLD and improper dynamic behavior of the MBRS will cause serious consequences (Yang and Xie, 1997).

Previous researches on BLD in the MBRS focuses on the following: Rotor shaft system stability, the variations of bearing oil film coefficients, critical speed, vibration response caused by the changing of bearing load and elevation, as well as the bearing load monitoring, etc. For example, stability studies of a large complex rotor system have been done by Lund and Bansal with a transfer matrix method (Lund, 1974; Bansal, 1975). Other research has concentrated on the influence of the elevation effect on systems' stability and bearing load distribution (Huang, 1987; Ding, 1993; Liu, 1995). The relationship between the displacement of the bearing journals and bearing force is demonstrated with the threemoments method (Li, 1988; Huang, 1993) and the transfer matrix method of Lumped Mass Mode(LMM) (Ni, 1989). The elevation sensitivity has been studied by Yang (Yang et al., 2000)

$1^{\#}, 2^{\#}, 3^{\#}, 4^{\#}, 11^{\#}-4$ pad tilting-pad bearing, $5^{\#}, 9^{\#}, 10^{\#}-2$ pad tilting-pad and a fixed pad bearing, $6^{\#}, 7^{\#}, 8^{\#}$ - round bearing

Fig. 1 MBRS diagram in a 600 MW turbine-generator set made in China
with the LMM model. Also, the authors studied the on-line monitoring methods and sensors in the MBRS (Yang and Xie, 1997 ; Yang et al., 1998 ; Yang et al., 2003).

The above-mentioned analyses and calculations need many branches of knowledge, and this process is complex. It may be difficult for general users of certain enterprises to master it within a short period of time. As a result, the cooperative research on product design based on the Internet, a distributed network of capturing information resource systems, is proposed here and this system is used to share available design resources with other remote customers in different places. This design concept is easy to put into practice and it will prove to be a good way to improve product quality, increase safety for the MBRS operation and result in a competent product. It is very important to analyze and diagnose any faults even remote ones of the MBRS.

Based on the above research, the following work has been done in this study: By the transfer matrix method of the Uniform Load Mode (ULM) (Xu, 1994), the loading sensitivity matrices were derived, and a new concept of BLS with load was proposed. The RC of the BLD and BLS based on the Internet is achieved by ASP technology. This research has the important significance of guaranteeing the safe operation of the MBRS, as well as on the sharing design resources and achieving a remote design and fault diagnosis.

## 2. Computational Mode of BLD

### 2.1 Rotor-bearing system

The MBRS constitutes a multi-support redundant beam system. Fig. 2 shows a typical por-


Fig. 2 Multi-support redundant beam system
tion of an ordinary multi-support and multi-span redundant beam system from the MBRS. It is composed of a rotor and bearings, it has $n$ supports and $n+1$ beam spans (simply supported span is formed between the two bearings, and the two extended spans are formed at two ends), and the distribute load $q$ can be decided with respect to the rotor diameter. For a MBRS with $n$ bearings, there are $n+1$ spans and $n+2$ nodes in the beam (Fig. 2). In the following sections, nodal numbers are expressed as $i=0,1,2, \cdots$, $n+1$, the span number $j=(1),(2), \cdots,(n+1)$, bearing (support) number $k=[1],[2], \cdots,[n]$.

### 2.2 Transfer matrix method

Usually, a span is composed of several shaft segments with different diameters (Fig. 3). For the $j^{\text {th }}$ span, it has $m$ shaft segments. Nodes are formed between the different shaft segments, there are $m+1$ nodes in the span as shown in Fig. 3, these nodes are expressed as $0,1,2, \cdots, m$. It should be mentioned that node 0 in span $j$ is the $(j-1)^{\text {th }}$ bearing. Meanwhile, the node $m$ in span $j$ is the $j^{\text {th }}$ node of bearing. The transformation for the conditional variable between the left and right nodes is established by the following proposed ULM transfer matrix method.

Fig. 4 shows the deformation and applied forces of the $i^{\text {th }}$ shaft on the $j^{\text {th }}$ beam span in $y o z$ plane. The $z$ axis is chosen as the baseline of the shaft. When the rotor is stationed inside the bearing, the center of the journal in the


Fig. $3 j^{\text {th }}$ beam span of the rotor


Fig. 4 Deformation and applied force of a shaft segment
bearing is chosen as static elevation; when the rotor is rotating, its $y$ coordinate represents a dynamic elevation. Uniform distribution loading $q_{i}(\mathrm{~N} / \mathrm{m})$ is applied to the $i^{\text {th }}$ shaft.

From the equilibrium of forces and moments, equation (1) is derived (Timoshenko, 1955), where, $Q_{i-1}$ and $M_{i-1}$ are the shear force and bending moment on the $(i-1)^{\text {th }}$ section. $Q_{i}$ and $M_{i}$ are the shear force and bending moment for the $i^{\text {th }}$ section. $y_{i}, \theta_{i}$ and, $y_{i-1}, \theta_{i-1}$ are the vertical deformation and deflection angles for the two sections, respectively. $E$ is Young's modulus, $J_{i}$ is the moment of inertia, $l_{i}$ is the length of the shaft $i$, and $q_{i}$ is the uniform load
$M_{i}=M_{i-1}-Q_{i-1} l_{i}+q_{i} l_{i}^{2} / 2$
$Q_{i}=Q_{i-1}-q_{i} l_{i}$
$y_{i}-y_{i-1}-\theta_{1-1} l_{i}$
$=M_{i} l_{i}^{2} /\left(2 E J_{i}\right)+Q_{i} l_{i}^{3} /\left(3 E J_{i}\right)+q_{i} l_{i}^{4} /\left(8 E J_{i}\right)$
$\theta_{i}-\theta_{i-1}$
$\left.=M_{i} l_{i} /\left(E J_{i}\right)+Q_{i} l_{i}^{2} /\left(2 E J_{i}\right)+q_{i} l_{i}^{2} /\left(6 E J_{i}\right) \quad\right)$
or in matrix form

$$
\begin{equation*}
\{Z\}_{i}=[\mathrm{N}]_{n}\{\mathrm{Z}\}_{i-1}+\{\mathbf{L}\}_{1} \tag{2}
\end{equation*}
$$

where,

$$
\begin{align*}
\{Z\}_{i} & =\{y, \theta, M, Q\}_{i}^{T} \\
\{L\}_{i} & =q_{i}\left\{l^{4} /(24 E J), l^{3} /(6 E J), l^{2} / 2,-l\right\}_{i}^{T} \\
{[N]_{i} } & =\left[\begin{array}{cccc}
1 & l & l^{2} /(2 E J) & -l 3 /(6 E J) \\
0 & 1 & l /(E J) & -l 2 /(2 E J) \\
0 & 0 & 1 & -l \\
0 & 0 & 0 & 1
\end{array}\right]_{i} \tag{3}
\end{align*}
$$

and superscript $T$ means transpose of a matrix.
Equation (2) is the transfer relationship between the $i-1$ node and $i$ node. For the whole segment of span, the following equation can be derived.

$$
\begin{aligned}
\{Z\}_{m} & =[N]_{m}\{Z\}_{m-1}+\{L\}_{m} \\
& =[N]_{m}[N]_{m-1}\{Z\}_{m-2}+[N]_{m}\{L\}_{m}+\{L\}_{m-1}(4) \\
& =[A]\{Z\}_{0}+[B]
\end{aligned}
$$

where

$$
\begin{aligned}
{[A]=} & {[N]_{m}[N]_{m-1} \cdots[N]_{2}[N]_{1} } \\
{[B]=} & \{L\}_{m}+[N]_{m}[L]_{m-1}+[N]_{m}[N]_{m-1}\{L\}_{m-2}+\cdots \\
& +[N]_{m}[N]_{m-1} \cdots[N]_{2}[L]_{1}
\end{aligned}
$$

from Fig. 3,

$$
\left.\begin{array}{l}
\{Z\}_{0}=\{Z\}_{j-1}^{R}  \tag{5}\\
\{Z\}_{m}=\{Z\}_{j}^{L}
\end{array}\right\}
$$

where the superscript $R$ means Right, $L$ means Left.

The transfer matrix between the $j-1^{\text {th }}$ node and $j^{\text {th }}$ node is defined by equations (4) and (5),

$$
\begin{equation*}
\{Z\}_{j}^{L}=[A]_{j}\{Z\}_{j-1}^{R}+[B]_{j} \tag{6}
\end{equation*}
$$

namely,

$$
\left(\begin{array}{c}
y  \tag{7}\\
\theta \\
M \\
Q
\end{array}\right)_{j}^{L}=\left[\begin{array}{cccc}
1 & a_{12} & a_{13} & a_{14} \\
0 & 1 & a_{23} & a_{24} \\
0 & 0 & 1 & a_{34} \\
0 & 0 & 0 & 1
\end{array}\right]_{j}\left(\begin{array}{c}
y \\
\theta \\
M \\
Q
\end{array}\right)_{j-1}^{R}+\left(\begin{array}{l}
b_{1} \\
b_{2} \\
b_{3} \\
b_{4}
\end{array}\right)_{j}
$$

where

$$
\begin{equation*}
a_{34}=-a_{12}, a_{13}-a_{24}=a_{12} a_{23} \tag{8}
\end{equation*}
$$

and

$$
\begin{equation*}
y_{j}^{L}=y_{j}^{R}=y_{j}, Q_{j}^{L}=Q_{j}^{R}=\theta_{j} \tag{9}
\end{equation*}
$$

By equations (7), (8), (9)

$$
\begin{align*}
& \left.\left|\begin{array}{l}
M \\
Q
\end{array}\right|_{j}^{2}=\left[\begin{array}{ll}
c_{11} & c_{12} \\
c_{21} & c_{22} \\
l_{2}
\end{array}\right]\left(\begin{array}{l}
y \\
\theta
\end{array}\right]_{j}-\left[\begin{array}{ll}
s_{11} & s_{12} \\
s_{21} & s_{22}
\end{array}\right]_{j}\left[\begin{array}{l}
y \\
\theta
\end{array}\right]_{j-1}+\left(\begin{array}{l}
b_{3} \\
b_{4}
\end{array}\right]_{j}-\left[\begin{array}{ll}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{array}\right]_{j}\left[\begin{array}{l}
b_{1} \\
b_{2}
\end{array}\right]_{j}\right\} \\
& \left.\left\{\left.\begin{array}{c}
M \\
Q
\end{array}\right|_{j} ^{R}=\left[\begin{array}{ll}
e_{11} & e_{12} \\
e_{21} & e_{22}
\end{array}\right],\left.\begin{array}{l}
y \\
\theta
\end{array}\right|_{j+1}-\left[\begin{array}{ll}
h_{11} & h_{12} \\
h_{21} & h_{2}
\end{array}\right]_{j+1}\left(\begin{array}{l}
y \\
\theta
\end{array}\right]_{j}-\left[\begin{array}{ll}
e_{11} & e_{12} \\
e_{21} & \left.e_{22}\right]_{j+1}
\end{array}\right]_{j 2}^{b_{1}} b_{2}\right]_{j+1}\right]_{j} \tag{10}
\end{align*}
$$

where,

$$
\begin{gathered}
{\left[\begin{array}{ll}
e_{11} & e_{12} \\
e_{21} & e_{22}
\end{array}\right]_{j}=\left[\begin{array}{ll}
a_{13} & e_{14} \\
e_{23} & e_{24}
\end{array}\right]^{-1}} \\
{\left[\begin{array}{ll}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{array}\right]_{j}=\left[\begin{array}{ll}
e_{11} & e_{12} \\
e_{21} & e_{22}
\end{array}\right]_{j}\left[\begin{array}{cc}
1 & a_{12} \\
0 & 1
\end{array}\right]_{j}} \\
{\left[\begin{array}{ll}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{array}\right]=\left[\begin{array}{ll}
1 & a_{34} \\
0 & 1
\end{array}\right]_{j}\left[\begin{array}{ll}
e_{11} & e_{12} \\
e_{21} & e_{22}
\end{array}\right]_{j}} \\
{\left[\begin{array}{ll}
s_{11} & s_{12} \\
s_{21} & s_{22}
\end{array}\right]_{j}=\left[\begin{array}{ll}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{array}\right]_{j}\left[\begin{array}{cc}
1 & a_{12} \\
0 & 1
\end{array}\right]_{j}=\left[\begin{array}{ll}
e_{22} & e_{12} \\
e_{21} & e_{11}
\end{array}\right]_{j}}
\end{gathered}
$$

### 2.3 Bearing load calculation

$P_{j}$ is the interactive force applied by the $j^{\text {th }}$ bearing on the shaft, namely, the vertical force applied by the $j^{\text {th }}$ bearing on the shaft. The force equilibrium in the vertical direction at $j^{\text {th }}$ bearing requires:

$$
\begin{equation*}
\binom{M}{Q}_{j}^{L}-\binom{M}{Q}_{j}^{R}=-\binom{0}{P}_{j} \tag{11}
\end{equation*}
$$

For the boundary condition shown in Fig. 2, (12) should be satisfied

$$
\begin{equation*}
\binom{M}{Q}_{0}^{R}=\binom{M}{Q}_{n+1}^{L}=\binom{0}{0} \tag{12}
\end{equation*}
$$

by inserting equation (10) into (11) and (12), the following two matrix equations can be obtained

$$
\begin{gather*}
{\left[K_{11}\right]\{Y\}+\left[K_{12}\right]\{\theta\}=-\{F\}}  \tag{13}\\
{\left[K_{21}\right]\{Y\}+\left[K_{22}\right]\{\theta\}=-\{P\}-\{S\}} \tag{14}
\end{gather*}
$$

Where,

$$
\begin{aligned}
& \left(\begin{array}{l}
f \\
s
\end{array}\right]_{j}=\left[\begin{array}{l}
b_{3} \\
b_{4}
\end{array}\right]_{j}-\left[\begin{array}{ll}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{array}\right]_{j}\left[\begin{array}{l}
b_{1} \\
b_{2}
\end{array}\right]_{j}+\left[\begin{array}{ll}
e_{11} & e_{12} \\
e_{21} & e_{22}
\end{array}\right]_{j}\left[\begin{array}{l}
b_{1} \\
b_{2}
\end{array}\right]_{j+1} \\
& j=2,3, \cdots, n-1 \\
& \binom{f}{s}_{1}=\left[\begin{array}{l}
b_{3} \\
b_{4}
\end{array}\right)_{1}+\left[\begin{array}{ll}
e_{11} & e_{12} \\
e_{21} & e_{22}
\end{array}\right]_{2}\left[\begin{array}{l}
b_{1} \\
b_{2}
\end{array}\right]_{2} \\
& \binom{f}{s}_{n}=\binom{b_{3}}{b_{4}}_{n}+\left[\begin{array}{cc}
1 & -a_{34} \\
0 & 1
\end{array}\right]_{n+1}\binom{b_{3}}{b_{4}}_{n+1} \\
& \{Y\}=\left\{y_{1}, y_{2}, \cdots, y_{n}\right\} ;\{\theta\}=\left\{\theta_{1}, \theta_{2}, \cdots, \theta_{n}\right\} \text {; } \\
& \{F\}=\left\{f_{1}, f_{2}, \cdots, f_{n}\right\} ;\{S\}=\left\{s_{1}, s_{2}, \cdots, s_{n}\right\} ; \\
& \{P\}=\left\{p_{1}, p_{2}, \cdots, p_{n}\right\}
\end{aligned}
$$

Combing equations (13), (14), a simplified equation can be reached

$$
\begin{equation*}
\{P\}=[K]\{F\}-\{S\}+[T]\{Y\} \tag{15}
\end{equation*}
$$

Where,

$$
\begin{gathered}
{[K]=\left[K_{22}\right]\left[K_{12}\right]^{-1}} \\
{[T]=\left[K_{22}\right]\left[K_{12}\right]^{-1}\left[K_{11}\right]-\left[K_{21}\right]} \\
{\left[\begin{array}{ccc}
h_{11}^{(2)} & -e_{11}^{(2)} \\
-e_{22}^{(2)} & h_{11}^{(3)}+c_{11}^{(2)}-e_{11}^{(3)} \\
\ddots & \\
& -e_{22}^{(n-1)} h_{11}^{(n)}+c_{11}^{(n-1)} & -e_{11}^{(n)} \\
& -e_{22}^{(n)} & c_{11}^{(n)}
\end{array}\right]_{n \times n}}
\end{gathered}
$$

$$
\left[K_{12}\right]=\left[\begin{array}{cccc}
h_{12}^{(2)} & -e_{12}^{(2)} & & \\
-e_{12}^{(2)} & h_{12}^{(3)}+c_{12}^{(2)} & -e_{12}^{(3)} & \\
& \ddots & & \\
& & -e_{12}^{(n-1)} & h_{12}^{(n)}+c_{12}^{(n-1)} \\
& & -e_{12}^{(n)} \\
& & & -e_{12}^{(n)} \\
& c_{12}^{(n)}
\end{array}\right]_{n \times n}
$$

$$
\left[K_{21}\right]=\left[\begin{array}{cccc}
h_{21}^{(2)} & -e_{21}^{(2)} & & \\
-e_{21}^{(2)} & h_{21}^{(3)}+c_{21}^{(2)} & -e_{21}^{(3)} & \\
& \ddots & & \\
& & -e_{21}^{(n-1)} & h_{21}^{(n)}+c_{21}^{(n-1)} \\
& & -e_{21}^{(n)} \\
& & & -e_{21}^{(n)}
\end{array} c_{21}^{(n)}\right]_{n \times n}
$$

and, $\left[K_{22}\right]=\left[K_{11}\right]^{T}, x, y=1$ or 2 in the $h_{x y}^{(j)}, e_{x y}^{(j)}$, $c_{x y}^{(j)}$, superscript $j$ is the corresponding parameters for the $j^{\text {th }}$ span.

Equation (15) can be used to estimate the vertical load $P_{i}$ on the shaft in a static equilibrium position or to solve a series of the shaft elevation $y_{i}$ by nonlinear iteration. By the same method, the matrix equation in the horizontal direction can be obtained. It has the same form as equation (15), but with $\{F\}=\{S\}=0$, due to the fact that there is no uniform load $q$ in horizontal direction, namely, $b_{1}=b_{2}=b_{3}=b_{4}=0$.

## 3. Analysis for Bearing Load Sensitivity

### 3.1 BLS to elevation variation

In equation (15), the first two terms on the Right Hand Side (RHS), $[K]\{F\}-[S]$ can be interpreted as the bearing loads when the elevation of all bearings within the MBRS are at the
same level or the bearing loads are at the same installation elevation with static shaft. The rest of the term on the RHS of equation (15) $[T]\{Y\}$, denotes the effect of elevation upon bearing loads. It should be noted that the loads are decided by relative elevation, and not by absolute elevation. Supposing that $i^{\text {th }}$ elevation $y_{i}$ varies, the other $n-1$ elevations remain unchanged, and the partial derivative on $y_{i}$ in equation (15) can be shown as

$$
\left(\begin{array}{c}
\frac{\partial p_{1}}{\partial y_{i}}  \tag{16}\\
\frac{\partial p_{2}}{\partial y_{i}} \\
\vdots \\
\frac{\partial p_{n}}{\partial y_{i}}
\end{array}\right]=\left[\begin{array}{cccc}
t_{11} & t_{12} & \cdots & t_{1 n} \\
t_{21} & t_{22} & \cdots & t_{2 n} \\
\vdots & & \vdots & \\
t_{n 1} & t_{n 2} & \cdots & t_{n n}
\end{array}\right]\left(\begin{array}{c}
0 \\
\vdots \\
1 \\
0 \\
\vdots \\
0
\end{array}\right)=\left(\begin{array}{c}
t_{i 1} \\
t_{22} \\
t_{i 3} \\
\vdots \\
t_{i n}
\end{array}\right)
$$

Thus, it is evident that the $i^{\text {th }}$ column of the matrix $[T]$ expresses the changes of every bearing load caused by the unit elevation change of the $i^{\text {th }}$ bearing. The matrix [ $T$ ] reflects the load sensitivity to elevation, and it is called an elevation sensitivity matrix. The element $t_{j i}$ in the matrix $[T]$ shows the load change of the $j^{\text {th }}$ bearing is related to the unit elevation change ( 1 m ) of the $i^{\text {th }}$ bearing. The elements on $j^{\text {th }}$ row in the matrix $[T]$ are elevation sensitivity coefficients of all bearings to the load on the $j^{\text {th }}$ bearing. Therefore, if the elevation variation for all bearings is obtained, all bearing loads can be obtained upon the linear combination of elevations.

### 3.2 BLS to load variation

It is hard to monitor the operating condition of the MBRS with the method of the monitoring bearing elevations. This is because the on site monitoring bearing elevation is very difficult. One study shows that a small variation in the elevation will result in a larger variation of BLD in the MBRS (Huang, 1993). To avoid this difficulty, the authors suggest an indirect monitoring scheme (Yang and Xie, 1997 ; Yang et al., 1998 ; Yang et al., 2003). In this scheme, the monitoring is done with an operating condition and the BLD of the MBRS is measured by monitoring
the bearing load. It is important that reciprocal relationships for the variations of loads between one bearing with the rest of the bearings in the MBRS be investigated.

Suppose that load $p_{i}$ and elevation $y_{i}$ vary on the $i^{\text {th }}$ bearing and the other $n-1$ bearing elevations remain constant, the varying load is, $\left(\partial p_{k} / \partial p_{i}\right),(k=1,2, \cdots, n)$, namely, the partial derivative on the $p_{i}$ equation on equation (15) gives

$$
\left(\begin{array}{c}
\frac{\partial p_{1}}{\partial p_{i}}  \tag{17}\\
\frac{\partial p_{i-1}}{\partial p_{i}} \\
1 \\
\frac{\partial p_{i+1}}{\partial p_{i}} \\
\vdots \\
\frac{\partial p_{n}}{\partial p_{i}}
\end{array}\right]=\left[\begin{array}{cccc}
t_{11} & t_{12} & \cdots & t_{1 n} \\
t_{21} & t_{22} & \cdots & t_{2 n} \\
\vdots & & \vdots & \\
t_{n 1} & t_{n 2} & \cdots & t_{n n}
\end{array}\right]\left(\begin{array}{c}
0 \\
\vdots \\
0 \\
\frac{\partial y_{i}}{\partial p_{i}} \\
0 \\
\vdots \\
0
\end{array}\right)
$$

Then, there are

$$
\begin{equation*}
\frac{\partial y_{i}}{\partial p_{i}}=\frac{1}{t_{i i}} \tag{18}
\end{equation*}
$$

and

$$
\begin{equation*}
\left(\frac{\partial p_{1}}{\partial p_{i}}, \frac{\partial p_{2}}{\partial p_{i}}, \cdots, \frac{\partial p_{n}}{\partial p_{i}}\right)^{T}=\frac{\left(t_{i 1}, t_{i 2}, \cdots, t_{i n}\right)^{T}}{t_{i i}} \tag{19}
\end{equation*}
$$

namely,

$$
[D]=\left[\begin{array}{cccc}
1 & \frac{t_{12}}{t_{22}} & \cdots & \frac{t_{1 n}}{t_{n n}}  \tag{20}\\
\frac{t_{21}}{t_{11}} & 1 & \cdots & \frac{t_{2 n}}{t_{n n}} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{t_{n 1}}{t_{11}} & \frac{t_{n 2}}{t_{22}} & \cdots & 1
\end{array}\right]
$$

Thus, it can be seen that all column elements $t_{j i}$ $(i=1,2, \cdots, n)$ in matrix $[T]$ are divided by the diagonal term $t_{i i}$ in matrix $[T]$, and matrix $[D]$ reflects the load sensitivity of the bearings with respect to the load variation. This is called the load sensitivity matrix. The element $\mathrm{d}_{j i}$ in matrix $[D]$ shows that the load variations for the $j^{\text {th }}$ bearing is caused by the unit load change (IN) of the $i^{\text {th }}$ bearing. The elements on the $j^{\text {th }}$ row in matrix [ $D$ ] are load sensitivity coefficients of all
bearings relative to the $j^{\text {th }}$ bearing load. Therefore, if the load variations for all bearings are obtained, all bearing loads will have the linear combination of the load variations.

### 3.3 Example

### 3.3.1 Symmetrical loading

Fig. 5 shows a symmetrical loading rotorbearing system that has 7 supports and 8 spans. The span length and corresponding diameters $\left(L_{(j)}, D_{(j)}\right)$ are $(0,0),(2,0.3),(2,0.3),(3,0.4)$, $(3,0.4),(2,0.3),(2,0.3)$ and $(0,0)$ meters, respectively, where the subscript $j=1,2, \cdots, 8$. The nonlinear iteration algorithm is implemented by Visual $\mathrm{C}++$ and MATLAB. The computational solution for this shaft is shown as
$[T]=\left[\begin{array}{ccccccc}-171.8 & 404.8 & -279.18 & 63.59 & -15.96 & 4.19 & -0.699 \\ 404.8 & -1173.4 & 1075.9 & -381.6 & 95.7 & -25.2 & 4.19 \\ -279.18 & 1075.9 & -1373.2 & 865.5 & -364.3 & 95.7 & -15.96 \\ 63.59 & -381.6 & 865.5 & -1095.2 & 865.5 & -381.6 & 63.59 \\ -1596 & 95.7 & -364.3 & 865.5 & -1373.2 & 1075.9 & -279.18 \\ 4.19 & -25.2 & 95.74 & -381.5 & 1075.9 & -1173.4 & 404.8 \\ -0.699 & 4.19 & -15.96 & 63.59 & -279.18 & 404.8 & -171.8\end{array}\right] \times 10^{5}(20)$

$$
[\mathrm{D}]=\left[\begin{array}{ccccccc}
1 & -0.345 & 0.207 & -0.058 & 0.012 & -0.004 & 0.004  \tag{21}\\
-2.354 & 1 & -0.783 & 0.348 & -0.070 & 0.021 & -0.024 \\
1.625 & -0.917 & 1 & -0.790 & 0.265 & -0.082 & 0.093 \\
-0.370 & 0.325 & -0.630 & 1 & -0.630 & 0.325 & -0.370 \\
0.093 & -0.082 & 0.265 & -0.790 & 1 & -0.917 & 1.625 \\
-0.024 & 0.021 & -0.070 & 0.348 & -0.783 & 1 & -2.354 \\
0.004 & -0.004 & 0.012 & -0.058 & 0.207 & -0.345 & 1
\end{array}\right]
$$

### 3.3.2 Unsymmetrical loading

The rotor-bearing system is the same as in Fig. 5 , but the span length and diameter $\left(L_{(j)}, D_{(j)}\right)$ $(j=1,2, \cdots, 8)$ are, $(0,0),(2,0.3),(2,0.3),(2$, $0.35),(2,0.35),(2,0.4),(2,0.4)$ and $(0,0)$ meters, respectively. The computational solution is shown as :


Fig. 5 Seven supports rotor-bearing system

$$
[T]=\left[\begin{array}{ccccccc}
-171.2 & 400.66 & -303.8 & 95.43 & -27.732 & 7.885 & -1.314  \tag{22}\\
400.66 & -1151 & 1196.5 & -572.58 & 166.39 & -47.3 & 7.8847 \\
-303.8 & 1196.5 & -1958.2 & 1699.7 & -831.1 & 236.29 & -39.38 \\
95.430 & -572.8 & 1699.7 & -2724.7 & 2328.8 & -992.1 & 165.3 \\
-27.73 & 166.39 & -831.08 & 2328.8 & -3449 & 2571.9 & -758.5 \\
7.8847 & -47.31 & 236.29 & -991.97 & 2571.9 & -2924 & 1147 \\
-1.314 & 7.8847 & -39.382 & 165.33 & -758.5 & 1147 & -521.04
\end{array}\right] \times 10^{5}
$$

$[\mathrm{D}]=\left[\begin{array}{ccccccc}1 & -0.348 & 0.155 & -0.035 & 0.008 & -0.003 & 0.003 \\ -2.341 & 1 & -0.611 & 0.210 & -0.048 & 0.016 & -0.015 \\ . .775 & -1.039 & 1 & -0.624 & 0.241 & -0.081 & 0.076 \\ -0.558 & 0.497 & -0.868 & 1 & -0.675 & 0.339 & -0.317 \\ 0.162 & -0.144 & 0.424 & -0.855 & 1 & -0.880 & 1.456 \\ -0.046 & 0.041 & -0.121 & 0.364 & -0.746 & 1 & -2.201 \\ 0.008 & -0.007 & 0.020 & -0.061 & 0.220 & -0.392 & 1\end{array}\right]$

### 3.4 Discussion and analysis

Matrices $[T]$ and $[D]$ for the symmetrical load rotor-bearing system are symmetrical, namely, $t_{j i}=t_{i j}, \quad d_{j i}=d_{n+1-j, n+1-l}$, where, $n$ is the number of bearings. For example, in equations (20), (21), $t_{32}=t_{23}$ means that load variations $(\mathrm{N})$ at the $3^{\text {rd }}$ bearing, which is caused by the unit elevation variation ( 1 m ) at the 2 nd bearing, equal the load variations ( N ) at the $2^{\text {nd }}$ bearing, which is caused by the unit elevation variation ( 1 m ) of the $3^{\text {rd }}$ bearing. When $n=7, d_{32}=d_{56}$ shows the load change ( N ) at the $3^{\text {rd }}$ bearing, which is caused by the unit load variations ( 1 N ) at the $2^{\text {nd }}$ bearing, this equals the load variations $(\mathrm{N})$ at the $5^{\text {th }}$ bearing, which is caused by the unit load variation ( 1 N ) at the $6^{\text {th }}$ bearing. There is an unsymmetrical property in the unsymmetrical loading rotor-bearing system, shown in equations (22), and (23).

By estimating the signs of elements in matrices $[T]$ and $[D]$, it can be concluded that the variations of load or elevation for one bearing affect the load direction of the adjacent bearings in negative ways. The variation of load or elevation at one bearing place within the MBRS will affect greatly the load of an adjacent bearing. The variation of the bearing load or elevation at one bearing will not certainly cause a large effect on itself. For example, the absolute value of element $d_{32}$ in equation (23), $\left|d_{32}\right|=1.039>\left|d_{22}\right|=1$, and the absolute value of element $t_{32}$ in equation
(22), $\left|t_{32}\right|=1196 .>\left|t_{22}\right|=1151$, show that the variation of load or elevation at the $3^{\text {rd }}$ bearing, which is caused by the variations of the load or elevation at the $2^{\text {nd }}$ bearing location, are bigger than the variation of the load and elevation at the $2^{\text {nd }}$ bearing itself.

The numerical value in matrix [ $T$ ] is usually large; it shows that a small variation of the relative elevation will result in large variations of the bearing load. By judging $d_{j i}>1$ or $<1$, it can be seen that matrix $[D]$ is clear in understanding the load effect caused by the variation of load at a certain bearing position to its neighbors.

## 4. RC Method

The idea of the RC method based on the Internet is in accordance with the working manner of the client/server. The clients understand the RC aim, algorithm and meaning of the parameters browsing the design website, then clients input the design parameters. The server obtains the request and activates the relevant processing program and output results to the customer website. The interface between the processing program and the HTML (Hypertext Markup Language) pages is connected with ASP, Java Applet and the CGI technique. The key to this technique is the data exchange between the underlying programs and the server application programs.

### 4.1 ASP method

ASP is a server script manager, it can be used to establish and operate a dynamic interactive Web server application program. The ASP script starts operating when the client browser requests the ASP file in the Web server, then the Web server calls the ASP and the ASP reads the required document, redirect the parameters filled in by client to the EXE and DLL files, and executes the script command and the EXE and DLL files. When computation is finished, the result is fed back to the server; from there the server sends back the calculated results into the Web page for browsing. The process is shown in Fig. 6. Since the script runs on the server rather


Fig. 6 ASP calling model
than the client-side and the Web page transmitted to the client browser is generated on the Web server, the browser can't deal with the script file. The standardized HTML will be transmitted to the browser after the Web server deals with all script files. Since only the calculated result is returned to the browser, the server-side script file is not easily copied and the client-side doesn't have to understand the script commands of the Web page being browsed. The ASP groupware is the Dynamic Link Library (DLL) file in essence ; theoretically, it can meet any needs from the client.

### 4.2 RC of DLL

It is an ideal choice to facilitate the concept of modern design by calling ASP in the form of DLL at different places. The ASP groupware can be designed by any High-level Language (HL). The special functions that are needed in the design can be encapsulated with function in advance and data exchanges can be achieved by files or a database in groupware. When the client proposes a request, the server runs the ASP programs. The data, which is filled by the client, will be saved in files or on databases and the ASP calls the groupware and functions, reads back data from files or databases and processing, and then the processed results are returned to the ASP files. Finally, the results designed in the special form are sent back to the client-side browser.

### 4.3 RC of EXE

Design computations in the engineering field may be called by the browser in places far away from home as long as it can be programmed in the form of EXE with HL. The server sends out the original data table to the client browser when the client needs to call the remote calculating programs, saves the data filled in by the client in files or on a database, and then runs the script file. The EXE reads the data and deals with it, and then ASP reads the calculating results saved in files or on databases. The Web server transmits the standardized HTML. (the results) to the client browser after all design computation tasks have been completed. The EXE and DDL methods are in accordance with each other.

### 4.4 RC of BLD and BLS

The computational model of the BLD and BLS in the MBRS is complied in FORTRAN, VC and MATLAB. The groupware includes 4 ASP files, 4 HTML flies and I DLL file.

The 4 ASP files are used to input and output data. Fig. 7 shows the RC output of the BLD of 7 support rotor systems (for example 3.3.1 Symmetrical loading) ; the first column shows the bearing loads, the second shows the relevant eccentricities, and the third shows the relevant off normal angles. Fig. 8 shows the RC results of the BLS matrix $[D]$ in the system. Four HTML files are used to introduce the bearing rotor system, which includes the parameter filling method and calculation results. The DLL file is used to link dynamically the computing process.

### 4.5 Discussion

Modern machine designs based on the Internet are a natural product and a developing tendency in the information age. It closely brings mechanical engineering techniques, network and computer techniques together. The RC of BLD and BLS in the MBRS provide a knowledge-acquiring platform to share technology and resources for the MBRS manufacturers, factories that use the MBRS and scientists in this field. This is necessary to boost product quality and competition capability.


Fig. 7 RC output of the BLD matrix


Fig. 8 RC output of BLS matrix [D]

The above-mentioned RC programs have been established on the Modern Design and Research Network web site (http://mdesign.tyut.edu.cn). The end user is only required to fill in some primary design data. Design solutions from different places can easily be obtained.

## 5. Conclusions

This research shows that the ideas of BLS are easy, new and practical in order to investigate the BLD in MBRS. The matrices [T] and $[D]$ of the symmetrical loading bearing-rotor system are symmetrical, namely, $t_{j i}=t_{i j}, d_{j i}=$ $d_{n+1-j, n+1-i}$. There is an unsymmetrical property in the unsymmetrical loading rotor-bearing system.

This study shows the following: The direction
of the bearing load or load induced by varying the elevation between adjacent bearing is affected in a negative way to the signs of terms at matrices $[T]$ and $[D]$. The variation of the load or elevation at one bearing place within the MBRS will affect the load of an adjacent bearing greatly.
Terms in the matrix $[T]$ are usually large; previous studies show that a small change in the relative elevation will cause large variations in the bearing load. By judging the matrix $[D]$, it will be easier for us to understand the load variation relations between one bearing and its neighbors.

The RC model and method based on the Internet are suitable. The RC of BLD and BLS in the MBRS are carried out.

## Acknowledgment

This work was supported by Shanxi Natural Science Foundation of China (SNSFC) (item number: 20001040) and the Korea Science and Engineering Foundation (KOSEF).

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[^0]:    * Corresponding Author,

    E-mail : yangzj@tyut.deu.cn
    TEL: +82-53-950-5577. +86-351-6014736
    FAX : +82-53-950-6588, +86 -351-6018320
    College of Mechanical Engineering, Taiyuan University of Technology, Taiyuan, 030024, China. (Manuscript
    Received August 19, 2003: Revised February 28, 2004)

